Find x: $x^x = y$,where y is a natural number larger than 0.

Hint: $x = e^{lnx}$, **use the Lambert W Function.** [1]

What is a Lambert W Function? Let

$$f(x) = xe^x$$

$$W(x) = f^{-1}(x)$$

Hence, $W(xe^x) = f^{-1}(f(x)) = x$

Solution:

$$(lnx)x = lny \tag{1}$$

$$(lnx)e^{lnx} = lny \tag{2}$$

$$W[(lnx)e^{lnx}] = W(lny) \tag{3}$$

$$lnx = W(lny) \tag{4}$$

$$e^{lnx} = e^{W(lny)} \tag{5}$$

$$x = e^{W(lny)} \tag{6}$$

When $y = 2$, put this in Wolfram Alpha, type Productlog instead of W. The result would be close to 1.56

---

[1]It can be used to derive Wien's displacement law. More in Astrophysics Notes!